Package ‘SubTS’

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Type Package

Title Positive Tempered Stable Distributions and Related Subordinators

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Description Contains methods for the simulation of positive tempered stable distributions and related subordinators. Including classical tempered stable, rapidly deceasing tempered stable, truncated stable, truncated tempered stable, generalized Dickman, truncated gamma, generalized gamma, and p-gamma. For details, see Dassios et al (2019) <doi:10.1017/jpr.2019.6>, Dassios et al (2020) <doi:10.1145/3368088>, Grabchak (2021) <doi:10.1016/j.spl.2020.109015>.

Suggests statmod

Imports copula, gsl, stats, tweedie

License GPL (>= 3)

NeedsCompilation yes

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`SubTS-package` *Positive Tempered Stable Distributions and Related Subordinators*

Description

Contains methods for the simulation of positive tempered stable distributions and related subordinators. Including classical tempered stable, rapidly deceasing tempered stable, truncated stable, truncated tempered stable, generalized Dickman, truncated gamma, generalized gamma, and p-gamma. For details, see Dassios et al (2019) <doi:10.1017/jpr.2019.6>, Dassios et al (2020) <doi:10.1145/3368088>, Grabchak (2021) <doi:10.1016/j.spl.2020.109015>.

Details

The DESCRIPTION file:

Package:	SubTS
Type:	Package
Title:	Positive Tempered Stable Distributions and Related Subordinators
Version:	1.0
Date:	2023-02-04
Authors@R:	c(person("Michael", "Grabchak", role = c("aut", "cre"), email = "mgrabcha@uncc.edu"), person("Lijuan", "Ca
Description:	Contains methods for the simulation of positive tempered stable distributions and related subordinators. Includ
Suggests:	statmod
Imports:	copula, gsl, stats, tweedie
License:	GPL (>= 3)
Author:	Michael Grabchak [aut, cre], Lijuan Can [aut]
Maintainer:	Michael Grabchak <mgrabcha@uncc.edu>

Index of help topics:

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dGGa                    Pdf of the generalized gamma distribution
dSubCTS                 PDF of CTS subordinator
```

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simTandW	Simulation of hitting time and overshoot.

Author(s)

NA

Maintainer: NA

References

A. Dassios, Y. Qu, J.W. Lim (2019). Exact simulation of generalised Vervaat perpetuities. Journal of Applied Probability, 56(1):57-75.

A. Dassios, Y. Qu, J.W. Lim (2020). Exact simulation of a truncated Levy subordinator. ACM Transactions on Modeling and Computer Simulation, 30(10), 17.

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
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rPRDTS(20, 2, 1, 0, 2)
rPRDTS(20, 2, 1, -.7, 2)
rDickman(10, 1)
rTrunGamma(10, 2, 1)
rPGamma(20, 2, 2, 2)
rTrunS(10, 2, .6)
rTrunTS(10, 2, 2, .6)
```

dF1 *Pdf for f_1*

Description

Evaluates the pdf f_1(x) intruduced in Grabchak (2021).

Usage

```
dF1(x, a, p)
```

Arguments

x	Vector of real numbers.
a	Parameter >=0.
p	Parameter >1.

Details

Evaluates the pdf

f_1(x) = exp(-x^p)*x^(-1-a)/K_1, x>1

where K_1 is a normalizing constant. This is distribution is needed to simulate p-RDTS random variables.

Value

Returns a vector of real numbers corresponding to the values of f_1(x).

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
x = (10:20)/10
dF1(x, .5, 2)
```

`dF2` *Pdf for f_2*

Description

Evaluates the pdf f_2(x) intruduced in Grabchak (2021).

Usage

```
dF2(x, a, p)
```

Arguments

`x`	Vector of real numbers.
`a`	Parameter in [0,1).
`p`	Parameter >1.

Details

Evaluates the pdf

f_2(x) = (exp(-x^p) - exp(-x))*x^(-1-a)/K_2, 0<x<1

where K_2 is a normalizing constant. This distribution is needed to simulate p-RDTS random variables.

Value

Returns a vector of real numbers corresponding to the values of f_2(x).

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
x = (0:10)/10
dF2(x, .5, 1.5)
```

`dGGa` *Pdf of the generalized gamma distribution*

Description

Evaluates the pdf of the generalized gamma distribution.

Usage

```
dGGa(x, a, p, b)
```

Arguments

`x`	Vector of real numbers.
`a`	Parameter >0.
`p`	Parameter >0.
`b`	Parameter >0.

Details

Evaluates the pdf of the generalized gamma distribution with density

g(x) = exp(-b*x^p)*x^(a-1)/K_3, x>0,

where K_3 is a normalizing constant. This distribution is needed to simulate p-RDTS random variables with negative alpha values.

Value

Returns a vector of real numbers corresponding to the values of g(x).

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

E.W. Stacy (1962) A generalization of the gamma distribution. Annals of Mathematical Statistics, 33(3):1187-1192.

Examples

```
x = (0:20)/10
dGGa(x, 2.5, 1.5, 3.1)
```

`dSubCTS` *PDF of CTS subordinator*

Description

Evaluates the pdf of the classical tempered stable (CTS) subordinator. When alpha=0 this is the pdf of the gamma distribution.

Usage

```
dSubCTS(x, alpha, c, ell)
```

Arguments

`x`	Vector of real numbers.
`alpha`	Parameter in [0,1).
`c`	Parameter >0
`ell`	Tempering parameter >0

Details

Returns the pdf of a classical tempered stable subordinator. The distribution has Laplace transform

L(z) = exp(c int_0^infty (e^(-xz)-1)e^(-x/ell) x^(-1-alpha) dx), z>0

and Levy measure

M(dx) = c e^(-x/ell) x^(-1-alpha) 1(x>0)dx.

Value

Returns a vector of real numbers corresponding to the values of pdf.

Note

Uses the method dtweedie in the Tweedie package.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
x = (0:20)/10
dSubCTS(x, .5, 1, 1.5)
```

getk1 *Constant K_1*

Description

Evaluates the constant K_1, which is the normalizing constant for f_1.

Usage

```
getk1(alpha, p)
```

Arguments

`alpha`	Parameter >=0.
`p`	Parameter >1.

Details

Evaluates

K_1 = int_1^infty exp(-x^p)*x^(-1-alpha) dx.

This is needed to simulate p-RDTS random variables.

Value

Returns a positive real number.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
getk1(1.5,2.5)
```

getk2 *Constant K_2*

Description

Evaluates the constant K_2, which is the normalizing constant for f_2.

Usage

```
getk2(alpha, p)
```

Arguments

alpha	Parameter in [0,1).
p	Parameter >1.

Details

Evaluates

K_2 = int_0^1 (exp(-x^p) - exp(-x))*x^(-1-alpha) dx.

This is needed to simulate p-RDTS random variables.

Value

Returns a positive real number.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
getk2(0.5,2.5)
```

`rDickman` *Simulation from the generalized Dickman distribution*

Description

Simulates from the generalized Dickman distribution using Algorithm 3.1 in Dassios, Qu, and Lim (2019).

Usage

```
rDickman(n, t, b = 1)
```

Arguments

`n`	Number of observations.
`t`	Parameter > 0.
`b`	Parameter > 0.

Details

Simulates from the generalized Dickman distribution by using Algorithm 3.1 in Dassios, Qu, and Lim (2019). This distribution has Laplace transform

L(z) = exp(t int_0^b (e^(-xz)-1) x^(-1) dx), z>0

and Levy measure

M(dx) = t x^(-1) 1(0<x<b) dx.

When b=1 and t=1, this is the Dickman distribution.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

A. Dassios, Y. Qu, J.W. Lim (2019). Exact simulation of generalised Vervaat perpetuities. Journal of Applied Probability, 56(1):57-75.

M. Penrose and A. Wade (2004). Random minimal directed spanning trees and Dickman-type distributions. Advances in Applied Probability, 36(3):691-714.

Examples

```
rDickman(10, 1)
```

rF1 *Simulation from f_1*

Description

Simulates from the pdf f_1(x) intruduced in Grabchak (2021).

Usage

```
rF1(n, a, p)
```

Arguments

n	Number of observations.
a	Parameter >=0.
p	Parameter >1.

Details

Uses Algorithm 1 in Grabchak (2021) to simulate from the pdf

f_1(x) = exp(-x^p)*x^(-1-a)/K_1, x>1,

where K_1 is a normalizing constant. This is needed to simulate p-RDTS random variables.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
rF1(10, .7, 2.5)
```

rF2 — *Simulation from f_2*

Description

Simulates from the pdf f_2(x) intruduced in Grabchak (2021).

Usage

```
rF2(n, a, p)
```

Arguments

n	Number of observations.
a	Parameter in [0,1).
p	Parameter >1.

Details

Uses Algorithm 2 in Grabchak (2021) to simulate from the pdf

f_2(x) = (exp(-x^p) - exp(-x))*x^(-1-a)/K_2, 0<x<1

where K_2 is a normalizing constant. This is needed to simulate p-RDTS random variables.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
rF2(10, .7, 2.5)
```

`rGGa` *Simulates from the generalized gamma distribution*

Description

Simulates from the generalized gamma distribution.

Usage

```
rGGa(n, a, p, b)
```

Arguments

n	Number of observations.
a	Parameter >0.
p	Parameter >0.
b	Parameter >0.

Details

Simulates from the generalized gamma distribution with density

g(x) = exp(-b*x^p)*x^(a-1)/K_3, x>0,

where K_3 is a normalizing constant. The mathodology is explained in Section 4 of Grabchak (2021). This distribution is needed to simulate p-RDTS random variables with negative alpha values.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

E.W. Stacy (1962) A generalization of the gamma distribution. Annals of Mathematical Statistics, 33(3):1187-1192.

Examples

```
rGGa(20, .5, 2, 2)
```

`rPGamma` *Simulation from p-gamma distributions.*

Description

Simulates from p-gamma distributions. These are p-RDTS distributions with alpha=0.

Usage

```
rPGamma(n, t, mu, p, step = 1)
```

Arguments

`n`	Number of observations.
`t`	Parameter >0.
`mu`	Parameter >0.
`p`	Parameter >1.
`step`	Tuning parameter. The larger the step, the slower the rejection sampling, but the fewer the number of terms. See Hoefert (2011) or Section 4 in Grabchak (2019).

Details

Uses Theorem 1 in Grabchak (2021) to simulate from a p-Gamma distribution. This distribution has Laplace transform

L(z) = exp(t int_0^infty (e^(-xz)-1)e^(-(mu*x)^p) x^(-1) dx), z>0

and Levy measure

M(dx) = t e^(-(mu*x)^p) x^(-1) 1(x>0)dx.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2019). Rejection sampling for tempered Levy processes. Statistics and Computing, 29(3):549-558

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

M. Hofert (2011). Sampling exponentially tilted stable distributions. ACM Transactions on Modeling and Computer Simulation, 22(1), 3.

Examples

```
rPGamma(20, 2, 2, 2)
```

---

rPRDTS *Simulation from p-RDTS distributions.*

Description

Simulates from p-rapidly decreasing tempered stable (p-RDTS) distributions.

Usage

```
rPRDTS(n, t, mu, alpha, p, step = 1)
```

Arguments

n	Number of observations.
t	Parameter >0.
mu	Parameter >0.
alpha	Parameter in (-infty,1)
p	Parameter >1 if 0<=alpha<1, >0 if alpha<0.
step	Tuning parameter. The larger the step, the slower the rejection sampling, but the fewer the number of terms. See Hoefert (2011) or Section 4 in Grabchak (2019).

Details

Simulates from a p-RDTS distribution. When alpha >=0, uses Theorem 1 in Grabchak (2021) and when alpha<0 uses the method in Section 4 of Grabchak (2021). This distribution has Laplace transform

L(z) = exp(t int_0^infty (e^(-xz)-1)e^(-(mu*x)^p) x^(-1-alpha) dx), z>0

and Levy measure

M(dx) = t e^(-(mu*x)^p) x^(-1-alpha) 1(x>0)dx.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2019). Rejection sampling for tempered Levy processes. Statistics and Computing, 29(3):549-558

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

M. Hofert (2011). Sampling exponentially tilted stable distributions. ACM Transactions on Modeling and Computer Simulation, 22(1), 3.

Examples

```
rPRDTS(20, 2, 1, .7, 2)
rPRDTS(20, 2, 1, 0, 2)
rPRDTS(20, 2, 1, -.7, 2)
```

---

rSubCTS *Simulates of CTS subordinators*

Description

Simulates from classical tempered stable (CTS) distributions. When alpha=0 this is the gamma distribution.

Usage

```
rSubCTS(n, alpha, c, ell, method = NULL)
```

Arguments

n	Number of observations.
alpha	Parameter in [0,1).
c	Parameter >0
ell	Tempering parameter >0
method	Parameter used by retstable in the copula package. When NULL restable selects the best method.

Details

Simulates a CTS subordinator. The distribution has Laplace transform

L(z) = exp(c int_0^infty (e^(-xz)-1)e^(-x/ell) x^(-1-alpha) dx), z>0

and Levy measure

M(dx) = c e^(-x/ell) x^(-1-alpha) 1(x>0)dx.

Value

Returns a vector of n random numbers.

Note

Uses the method retstable in the copula package.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
rSubCTS(20, .7, 1, 1)
```

`rTrunGamma` *Simulation from the truncated gamma distribution*

Description

Simulates from the truncated gamma distribution.

Usage

```
rTrunGamma(n, t, mu, b = 1, step = 1)
```

Arguments

`n`	Number of observations.
`t`	Parameter > 0.
`mu`	Parameter > 0.
`b`	Parameter > 0.
`step`	Tuning parameter. The larger the step, the slower the rejection sampling, but the fewer the number of terms. See Hoefert (2011) or Section 4 in Grabchak (2019).

Details

Simulates from the truncated gamma distribution. This distribution has Laplace transform

L(z) = exp(t int_0^b (e^(-xz)-1) x^(-1)e^(-mu*x) dx), z>0

and Levy measure

M(dx) = t x^(-1) e^(-mu*x) 1(0<x<b) dx.

The simulation is performed by applying rejection sampling (Algorithm 4.4 in Dassios, Qu, Lim (2020)) to the generalized Dickman distribution. We simulate from the latter using Algorithm 3.1 in Dassios, Qu, Lim (2019).

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

A. Dassios, Y. Qu, J.W. Lim (2019). Exact simulation of generalised Vervaat perpetuities. Journal of Applied Probability, 56(1):57-75.

A. Dassios, Y. Qu, J.W. Lim (2020). Exact simulation of a truncated Levy subordinator. ACM Transactions on Modeling and Computer Simulation, 30(10), 17.

M. Grabchak (2019). Rejection sampling for tempered Levy processes. Statistics and Computing, 29(3):549-558

M. Hofert (2011). Sampling exponentially tilted stable distributions. ACM Transactions on Modeling and Computer Simulation, 22(1), 3.

Examples

```
rTrunGamma(10, 2, 1)
```

---

rTrunS *Simulation from the truncated stable distribution*

Description

Simulates from the truncated stable distribution.

Usage

```
rTrunS(n, t, alpha, b = 1, step = 1)
```

Arguments

n	Number of observations.
t	Parameter > 0.
alpha	Parameter in the open interval (0,1).
b	Parameter > 0.
step	Tuning parameter. The larger the step, the slower the rejection sampling, but the fewer the number of terms. See Hoefert (2011) or Section 4 in Grabchak (2019).

Details

Simulates from the truncated stable distribution using Algorithm 4.3 in Dassios, Qu, and Lim (2020). This distribution has Laplace transform

L(z) = exp(t * (alpha/Gamma(1-alpha)) * int_0^b (e^(-xz)-1) x^(-1-alpha) dx), z>0

and Levy measure

M(dx) = t * (alpha/Gamma(1-alpha)) * x^(-1-alpha) 1(0<x<b) dx.

Here Gamma() is the gamma function.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

A. Dassios, Y. Qu, J.W. Lim (2020). Exact simulation of a truncated Levy subordinator. ACM Transactions on Modeling and Computer Simulation, 30(10), 17.

M. Grabchak (2019). Rejection sampling for tempered Levy processes. Statistics and Computing, 29(3):549-558

M. Hofert (2011). Sampling exponentially tilted stable distributions. ACM Transactions on Modeling and Computer Simulation, 22(1), 3.

Examples

```
rTrunS(10, 2, .6)
```

`rTrunTS` *Simulation from the truncated tempered stable distribution.*

Description

Simulates from the truncated tempered stable distribution.

Usage

```
rTrunTS(n, t, mu, alpha, b = 1, step = 1)
```

Arguments

`n`	Number of observations.
`t`	Parameter > 0.
`mu`	Parameter > 0.
`alpha`	Parameter in the open interval (0,1).
`b`	Parameter > 0.
`step`	Tuning parameter. The larger the step, the slower the rejection sampling, but the fewer the number of terms. See Hoefert (2011) or Section 4 in Grabchak (2019).

Details

Simulates from the truncated stable distribution using Algorithm 4.3 in Dassios, Qu, and Lim (2020). This distribution has Laplace transform

L(z) = exp(t * (alpha/Gamma(1-alpha)) * int_0^b (e^(-xz)-1) x^(-1-alpha) e^(-mu*x) dx), z>0

and Levy measure

M(dx) = t * (alpha/Gamma(1-alpha)) * x^(-1-alpha) e^(-mu*x) 1(0<x<b) dx.

Here Gamma() is the gamma function.

Value

Returns a vector of n random numbers.

Author(s)

Michael Grabchak and Lijuan Cao

References

A. Dassios, Y. Qu, J.W. Lim (2020). Exact simulation of a truncated Levy subordinator. ACM Transactions on Modeling and Computer Simulation, 30(10), 17.

M. Grabchak (2019). Rejection sampling for tempered Levy processes. Statistics and Computing, 29(3):549-558

M. Hofert (2011). Sampling exponentially tilted stable distributions. ACM Transactions on Modeling and Computer Simulation, 22(1), 3.

Examples

```
rTrunTS(10, 2, 2, .6)
```

simCondS *Simulation from a conditioned stable distribution.*

Description

Implements Algorithm 4.2 in Dassios, Qu, and Lim (2020) to simulate from a stable distribution conditioned on an appropriate event.

Usage

```
simCondS(t, alpha)
```

Arguments

`t`	Parameter > 0.
`alpha`	Parameter in the open interval (0,1).

Details

Implements Algorithm 4.2 in Dassios, Qu, and Lim (2020) to simulate from a stable distribution conditioned on an appropriate event. There are some typos in this algorithm, which are corrected in Grabchak (2021). These random variables are needed to simulate truncated stable, truncated tempered stable, and p-RDTS random variables.

Value

Returns one random number.

Author(s)

Michael Grabchak and Lijuan Cao

References

A. Dassios, Y. Qu, J.W. Lim (2020). Exact simulation of a truncated Levy subordinator. ACM Transactions on Modeling and Computer Simulation, 30(10), 17.

M. Grabchak (2021). An exact method for simulating rapidly decreasing tempered stable distributions. Statistics and Probability Letters, 170: Article 109015.

Examples

```
simCondS(2, .7)
```

`simTandW` *Simulation of hitting time and overshoot.*

Description

Simulates the hitting time T and the overshoot W of a stable process by implimenting Algorithm 4.1 in Dassios, Qu, and Lim (2020). This is important for simulating other distribution.

Usage

```
simTandW(alpha)
```

Arguments

`alpha`	Parameter in the open interval (0,1).

Value

Returns one pair of random numbers. The first is T and the second is W.

Author(s)

Michael Grabchak and Lijuan Cao

References

A. Dassios, Y. Qu, J.W. Lim (2020). Exact simulation of a truncated Levy subordinator. ACM Transactions on Modeling and Computer Simulation, 30(10), 17.

Examples

```
simTandW(.6)
```

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