

Package ‘mgc’

June 23, 2020

Type Package

Title Multiscale Graph Correlation

Version 2.0.2

Date 2020-06-20

Maintainer Eric Bridgeford <ericwb95@gmail.com>

Description Multiscale Graph Correlation (MGC) is a framework developed by Vogelstein et al. (2019) <DOI:10.7554/eLife.41690> that extends global correlation procedures to be multiscale; consequently, MGC tests typically require far fewer samples than existing methods for a wide variety of dependence structures and dimensionalities, while maintaining computational efficiency. Moreover, MGC provides a simple and elegant multiscale characterization of the potentially complex latent geometry underlying the relationship.

Depends R (>= 3.4.0)

Imports stats, MASS, abind, boot, energy, raster

URL <https://github.com/neurodata/r-mgc>

Suggests testthat (>= 2.1.0), ggplot2, reshape2, knitr, rmarkdown

License GPL-2

Encoding UTF-8

LazyData true

RoxygenNote 7.0.2

VignetteBuilder knitr

NeedsCompilation yes

Author Eric Bridgeford [aut, cre],
Censheng Shen [aut],
Shangsi Wang [aut],
Joshua Vogelstein [ths]

Repository CRAN

Date/Publication 2020-06-23 12:50:18 UTC

R topics documented:

ConnCompLabel	2
discr.sims.cross	3
discr.sims.exp	4
discr.sims.fat_tails	5
discr.sims.linear	6
discr.sims.radial	7
discr.stat	8
discr.test.one_sample	10
discr.test.two_sample	12
mgc.dist.xfm	14
mgc.distance	15
mgc.ksample	16
mgc.localcorr	17
mgc.localcorr.driver	19
mgc.sims.2ball	20
mgc.sims.2sphere	21
mgc.sims.cubic	21
mgc.sims.exp	23
mgc.sims.joint	24
mgc.sims.linear	25
mgc.sims.quad	26
mgc.sims.spiral	27
mgc.sims.step	28
mgc.sims.ubern	29
mgc.sims.wshape	30
mgc.stat	31
mgc.test	33
Index	36

 ConnCompLabel

Connected Components Labelling – Unique Patch Labelling

Description

ConnCompLabel is a 1 pass implementation of connected components labelling. Here it is applied to identify disjunct patches within a distribution.

The raster matrix can be a raster of class 'asc' (adehabitat package), 'RasterLayer' (raster package) or 'SpatialGridDataFrame' (sp package).

Usage

```
ConnCompLabel(mat)
```

Arguments

`mat` is a binary matrix of data with 0 representing background and 1 representing environment of interest. NA values are acceptable. The matrix can be a raster of class 'asc' (this & adehabitat package), 'RasterLayer' (raster package) or 'SpatialGridDataFrame' (sp package)

Value

A matrix of the same dim and class of `mat` in which unique components (individual patches) are numbered 1:n with 0 remaining background value.

Author(s)

Jeremy VanDerWal <jjvanderwal@gmail.com>

References

Chang, F., C.-J. Chen, and C.-J. Lu. 2004. A linear-time component-labeling algorithm using contour tracing technique. *Comput. Vis. Image Underst.* 93:206-220.

Examples

```
#define a simple binary matrix
tmat = { matrix(c( 0,0,0,1,0,0,1,1,0,1,
                 0,0,1,0,1,0,0,0,0,0,
                 0,1,NA,1,0,1,0,0,0,1,
                 1,0,1,1,1,0,1,0,0,1,
                 0,1,0,1,0,1,0,0,0,1,
                 0,0,1,0,1,0,0,1,1,0,
                 1,0,0,1,0,0,1,0,0,1,
                 0,1,0,0,0,1,0,0,0,1,
                 0,0,1,1,1,0,0,0,0,1,
                 1,1,1,0,0,0,0,0,0,1),nr=10,byrow=TRUE) }

#do the connected component labelling
ccl.mat = ConnCompLabel(tmat)
ccl.mat
image(t(ccl.mat[10:1,]),col=c('grey',rainbow(length(unique(ccl.mat))-1)))
```

Description

A function to simulate data with the same mean that spreads as class id increases.

Usage

```
discr.sims.cross(  
  n,  
  d,  
  K,  
  signal.scale = 10,  
  non.scale = 1,  
  mean.scale = 0,  
  rotate = FALSE,  
  class.equal = TRUE,  
  ind = FALSE  
)
```

Arguments

n	the number of samples.
d	the number of dimensions.
K	the number of classes in the dataset.
signal.scale	the scaling for the signal dimension. Defaults to 10.
non.scale	the scaling for the non-signal dimensions. Defaults to 1.
mean.scale	whether the magnitude of the difference in the means between the two classes. If a mean scale is requested, d should be at least > K.
rotate	whether to apply a random rotation. Defaults to TRUE.
class.equal	whether the number of samples/class should be equal, with each class having a prior of 1/K, or unequal, in which each class obtains a prior of k/sum(K) for k=1:K. Defaults to TRUE.
ind	whether to sample x and y independently. Defaults to FALSE.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)  
sim <- discr.sims.cross(100, 3, 2)
```

discr.sims.exp

Discriminability Exponential Simulation

Description

A function to simulate multi-class data with an Exponential class-mean trend.

Usage

```
discr.sims.exp(  
  n,  
  d,  
  K,  
  signal.scale = 1,  
  signal.lshift = 1,  
  non.scale = 1,  
  rotate = FALSE,  
  class.equal = TRUE,  
  ind = FALSE  
)
```

Arguments

n	the number of samples.
d	the number of dimensions. The first dimension will be the signal dimension; the remainders noise.
K	the number of classes in the dataset.
signal.scale	the scaling for the signal dimension. Defaults to 1.
signal.lshift	the location shift for the signal dimension between the classes. Defaults to 1.
non.scale	the scaling for the non-signal dimensions. Defaults to 1.
rotate	whether to apply a random rotation. Defaults to TRUE.
class.equal	whether the number of samples/class should be equal, with each class having a prior of $1/K$, or unequal, in which each class obtains a prior of $k/\sum(K)$ for $k=1:K$. Defaults to TRUE.
ind	whether to sample x and y independently. Defaults to FALSE.

Author(s)

Eric Bridgeford

discr.sims.fat_tails *Discriminability Spread Simulation*

Description

A function to simulate data with the same mean that spreads as class id increases.

Usage

```
discr.sims.fat_tails(  
  n,  
  d,  
  K,  
  signal.scale = 1,  
  rotate = FALSE,  
  class.equal = TRUE,  
  ind = FALSE  
)
```

Arguments

n	the number of samples.
d	the number of dimensions.
K	the number of classes in the dataset.
signal.scale	the scaling for the signal dimension. Defaults to 1.
rotate	whether to apply a random rotation. Defaults to TRUE.
class.equal	whether the number of samples/class should be equal, with each class having a prior of 1/K, or unequal, in which each class obtains a prior of $k/\sum(K)$ for $k=1:K$. Defaults to TRUE.
ind	whether to sample x and y independently. Defaults to FALSE.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)  
sim <- discr.sims.fat_tails(100, 3, 2)
```

discr.sims.linear *Discriminability Linear Simulation*

Description

A function to simulate multi-class data with a linear class-mean trend. The signal dimension is the dimension carrying all of the between-class difference, and the non-signal dimensions are noise.

Usage

```
discr.sims.linear(  
  n,  
  d,  
  K,  
  signal.scale = 1,  
  signal.lshift = 1,  
  non.scale = 1,  
  rotate = FALSE,  
  class.equal = TRUE,  
  ind = FALSE  
)
```

Arguments

n	the number of samples.
d	the number of dimensions. The first dimension will be the signal dimension; the remainders noise.
K	the number of classes in the dataset.
signal.scale	the scaling for the signal dimension. Defaults to 1.
signal.lshift	the location shift for the signal dimension between the classes. Defaults to 1.
non.scale	the scaling for the non-signal dimensions. Defaults to 1.
rotate	whether to apply a random rotation. Defaults to TRUE.
class.equal	whether the number of samples/class should be equal, with each class having a prior of $1/K$, or unequal, in which each class obtains a prior of $k/\text{sum}(K)$ for $k=1:K$. Defaults to TRUE.
ind	whether to sample x and y independently. Defaults to FALSE.

Author(s)

Eric Bridgeford

discr.sims.radial *Discriminability Radial Simulation*

Description

A function to simulate data with the same mean with radial symmetry as class id increases.

Usage

```
discr.sims.radial(  
  n,  
  d,  
  K,  
  er.scale = 0.1,  
  r = 1,  
  class.equal = TRUE,  
  ind = FALSE  
)
```

Arguments

n	the number of samples.
d	the number of dimensions.
K	the number of classes in the dataset.
er.scale	the scaling for the error of the samples. Defaults to 0.1.
r	the radial spacing between each class. Defaults to 1.
class.equal	whether the number of samples/class should be equal, with each class having a prior of 1/K, or unequal, in which each class obtains a prior of $k/\sum(K)$ for $k=1:K$. Defaults to TRUE.
ind	whether to sample x and y independently. Defaults to FALSE.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)  
sim <- discr.sims.radial(100, 3, 2)
```

discr.stat

Discriminability Statistic

Description

A function for computing the discriminability from a distance matrix and a set of associated labels.

Usage

```
discr.stat(
  X,
  Y,
  is.dist = FALSE,
  dist.xfm = mgc.distance,
  dist.params = list(method = "euclidean"),
  dist.return = NULL,
  remove.isolates = TRUE
)
```

Arguments

X	is interpreted as: a [n x d] data matrix X is a data matrix with n samples in d dimensions, if flag <code>is.dist=FALSE</code> . a [n x n] distance matrix X is a distance matrix. Use flag <code>is.dist=TRUE</code> .
Y	[n] a vector containing the sample ids for our n samples.
is.dist	a boolean indicating whether your X input is a distance matrix or not. Defaults to FALSE.
dist.xfm	if <code>is.dist == FALSE</code> , a distance function to transform X. If a distance function is passed, it should accept an [n x d] matrix of n samples in d dimensions and return a [n x n] distance matrix, which can be either the default output, an item castable to a distance matrix, or . See mgc.distance for details.
dist.params	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm</code> . Defaults to <code>list(method='euclidean')</code> .
dist.return	the return argument for the specified <code>dist.xfm</code> containing the distance matrix. Defaults to FALSE. <code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm</code> as the distance matrix. Should be an object castable to a [n x n] matrix. You can verify whether this is the case by looking at <code>as.matrix(do.call(dist.xfm, list(X, <trailing_ar</code> <code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm[[dist.return]]</code> as the distance matrix. Should be a [n x n] matrix.
remove.isolates	remove isolated samples from the dataset. Isolated samples are samples with only one instance of their class appearing in the Y vector. Defaults to TRUE.

Value

A list containing the following:

discr	the discriminability statistic.
rdf	the rdfs for each sample.

Details

For more details see the help vignette: `vignette("discriminability", package = "mgc")`

Author(s)

Eric Bridgeford

References

Eric W. Bridgeford, et al. "Optimal Decisions for Reference Pipelines and Datasets: Applications in Connectomics." Bioarxiv (2019).

Examples

```
sim <- discr.sims.linear(100, 10, K=2)
X <- sim$X; Y <- sim$Y
discr.stat(X, Y)$discr
```

discr.test.one_sample *Discriminability One Sample Permutation Test*

Description

A function that performs a one-sample test for whether the discriminability differs from random chance.

Usage

```
discr.test.one_sample(
  X,
  Y,
  is.dist = FALSE,
  dist.xfm = mgc.distance,
  dist.params = list(method = "euclidean"),
  dist.return = NULL,
  remove.isolates = TRUE,
  nperm = 500,
  no_cores = 1
)
```

Arguments

X	is interpreted as: a [n x d] data matrix X is a data matrix with n samples in d dimensions, if flag is.dist=FALSE. a [n x n] distance matrix X is a distance matrix. Use flag is.dist=TRUE.
Y	[n] a vector containing the sample ids for our n samples.
is.dist	a boolean indicating whether your X input is a distance matrix or not. Defaults to FALSE.

<code>dist.xfm</code>	if <code>is.dist == FALSE</code> , a distance function to transform X . If a distance function is passed, it should accept an $[n \times d]$ matrix of n samples in d dimensions and return a $[n \times n]$ distance matrix as the $\$D$ return argument. See mgc.distance for details.
<code>dist.params</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return</code>	the return argument for the specified <code>dist.xfm</code> containing the distance matrix. Defaults to <code>FALSE</code> .
	<code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm</code> as the distance matrix. Should be a $[n \times n]$ matrix.
	<code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm[[dist.return]]</code> as the distance matrix. Should be a $[n \times n]$ matrix.
<code>remove.isolates</code>	remove isolated samples from the dataset. Isolated samples are samples with only one instance of their class appearing in the Y vector. Defaults to <code>TRUE</code> .
<code>nperm</code>	the number of permutations to perform. Defaults to 500.
<code>no_cores</code>	the number of cores to use for permutation test. Defaults to 1.

Value

A list containing the following:

<code>stat</code>	the discriminability of the data.
<code>null</code>	the discriminability scores under the null, computed via permutation.
<code>p.value</code>	the pvalue associated with the permutation test.

Details

Performs a test of whether an observed discriminability is significantly different from chance, as described in Bridgeford et al. (2019). With \hat{D}_X the sample discriminability of X :

$$H_0 : D_X = D_0$$

and:

$$H_A : D_X > D_0$$

where D_0 is the discriminability that would be observed by random chance.

Author(s)

Eric Bridgeford

References

Eric W. Bridgeford, et al. "Optimal Decisions for Reference Pipelines and Datasets: Applications in Connectomics." *Bioarxiv* (2019).

Examples

```
## Not run:
require(mgc)
n = 100; d=5

# simulation with a large difference between the classes
# meaning they are more discriminable
sim <- discr.sims.linear(n=n, d=d, K=2, signal.lshift=10)
X <- sim$X; Y <- sim$Y

# p-value is small
discr.test.one_sample(X, Y)$p.value

## End(Not run)
```

discr.test.two_sample *Discriminability Two Sample Permutation Test*

Description

A function that takes two sets of paired data and tests of whether or not the data is more, less, or non-equally discriminable between the set of paired data.

Usage

```
discr.test.two_sample(
  X1,
  X2,
  Y,
  dist.xfm = mgc.distance,
  dist.params = list(method = "euclidian"),
  dist.return = NULL,
  remove.isolates = TRUE,
  nperm = 500,
  no_cores = 1,
  alt = "greater"
)
```

Arguments

X1	is interpreted as a [n x d] data matrix with n samples in d dimensions. Should NOT be a distance matrix.
X2	is interpreted as a [n x d] data matrix with n samples in d dimensions. Should NOT be a distance matrix.
Y	[n] a vector containing the sample ids for our n samples. Should be matched such that Y[i] is the corresponding label for X1[i,] and X2[i,].

<code>dist.xfm</code>	if <code>is.dist == FALSE</code> , a distance function to transform X. If a distance function is passed, it should accept an $[n \times d]$ matrix of n samples in d dimensions and return a $[n \times n]$ distance matrix as the <code>\$D</code> return argument. See mgc.distance for details.
<code>dist.params</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return</code>	the return argument for the specified <code>dist.xfm</code> containing the distance matrix. Defaults to <code>FALSE</code> . <code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm</code> as the distance matrix. Should be a $[n \times n]$ matrix. <code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm[[dist.return]]</code> as the distance matrix. Should be a $[n \times n]$ matrix.
<code>remove.isolates</code>	remove isolated samples from the dataset. Isolated samples are samples with only one instance of their class appearing in the Y vector. Defaults to <code>TRUE</code> .
<code>nperm</code>	the number of permutations for permutation test. Defaults to 500.
<code>no_cores</code>	the number of cores to use for the permutations. Defaults to 1.
<code>alt</code>	the alternative hypothesis. Can be that first dataset is more discriminable (<code>alt = 'greater'</code>), less discriminable (<code>alt = 'less'</code>), or just non-equal (<code>alt = 'neq'</code>). Defaults to "greater".

Value

A list containing the following:

<code>stat</code>	the observed test statistic. the test statistic is the difference in discriminability of X1 vs X2.
<code>discr</code>	the discriminabilities for each of the two data sets, as a list.
<code>null</code>	the null distribution of the test statistic, computed via permutation.
<code>p.value</code>	The p-value associated with the test.
<code>alt</code>	The alternative hypothesis for the test.

Details

A function that performs a two-sample test for whether the discriminability is different for that of one dataset vs another, as described in Bridgeford et al. (2019). With \hat{D}_{X_1} the sample discriminability of one approach, and \hat{D}_{X_2} the sample discriminability of another approach:

$$H_0 : D_{X_1} = D_{X_2}$$

and:

$$H_A : D_{X_1} > D_{X_2}$$

. Also implemented are tests of $<$ and \neq .

Author(s)

Eric Bridgeford

References

Eric W. Bridgeford, et al. "Optimal Decisions for Reference Pipelines and Datasets: Applications in Connectomics." Bioarxiv (2019).

Examples

```
## Not run:
require(mgc)
require(MASS)

n = 100; d=5

# generate two subjects truths; true difference btwn
# subject 1 (column 1) and subject 2 (column 2)
mus <- cbind(c(0, 0), c(1, 1))
Sigma <- diag(2) # dimensions are independent

# first dataset X1 contains less noise than X2
X1 <- do.call(rbind, lapply(1:dim(mus)[2],
  function(k) {mvrnorm(n=50, mus[,k], 0.5*Sigma)}))
X2 <- do.call(rbind, lapply(1:dim(mus)[2],
  function(k) {mvrnorm(n=50, mus[,k], 2*Sigma)}))
Y <- do.call(c, lapply(1:2, function(i) rep(i, 50)))

# X1 should be more discriminable, as less noise
discr.test.two_sample(X1, X2, Y, alt="greater")$p.value # p-value is small

## End(Not run)
```

mgc.dist.xfm

MGC Distance Transform

Description

Transform the distance matrices, with column-wise ranking if needed.

Usage

```
mgc.dist.xfm(X, Y, option = "mgc", optionRk = TRUE)
```

Arguments

X	[nxn] is a distance matrix
Y	[nxn] is a second distance matrix
option	is a string that specifies which global correlation to build up-on. Defaults to mgc. 'mgc' use the MGC global correlation. 'dcor' use the dcor global correlation.

'mantel' use the mantel global correlation.
 'rank' use the rank global correlation.

optionRk is a string that specifies whether ranking within column is computed or not. If option='rank', ranking will be performed regardless of the value specified by optionRk. Defaults to TRUE.

Value

A list containing the following:

A [nxn] the centered distance matrix for X.
 B [nxn] the centered distance matrix for Y.
 RX [nxn] the column-rank matrices of X.
 RY [nxn] the column-rank matrices of Y.

Author(s)

C. Shen

Examples

```
library(mgc)

n=200; d=2
data <- mgc.sims.linear(n, d)
Dx <- as.matrix(dist(data$X), nrow=n); Dy <- as.matrix(dist(data$Y), nrow=n)
dt <- mgc.dist.xfm(Dx, Dy)
```

mgc.distance	<i>Distance</i>
--------------	-----------------

Description

A function that returns a distance matrix given a collection of observations.

Usage

```
mgc.distance(X, method = "euclidean")
```

Arguments

X [n x d] a data matrix for d samples of d variables.
 method the method for computing distances. Defaults to 'euclidean'. See [dist](#) for details. Also includes a "ohe" option, which one-hot-encodes the matrix when computing distances.

Value

a [n x n] distance matrix indicating the pairwise distances between all samples passed in.

Author(s)

Eric Bridgeford

mgc.ksample

MGC K Sample Testing

Description

MGC K Sample Testing provides a wrapper for MGC Sample testing under the constraint that the Ys here are categorical labels with K possible sample ids. This function uses a 0-1 loss for the Ys (one-hot-encoding).

Usage

```
mgc.ksample(X, Y, mgc.opts = list(), ...)
```

Arguments

X	is interpreted as: a [n x d] data matrix X is a data matrix with n samples in d dimensions, if flag <code>is.dist.X=FALSE</code> . a [n x n] distance matrix X is a distance matrix. Use flag <code>is.dist.X=TRUE</code> .
Y	[n] the labels of the samples with K unique labels.
mgc.opts	Arguments to pass to MGC, as a named list. See mgc.test for details. Do not pass arguments for <code>is.dist.Y</code> , <code>dist.xfm.Y</code> , <code>dist.params.Y</code> , nor <code>dist.return.Y</code> , as they will be ignored.
...	trailing args.

Value

A list containing the following:

p.value	P-value of MGC
stat	is the sample MGC statistic within [-1, 1]
pLocalCorr	P-value of the local correlations by double matrix index
localCorr	the local correlations
optimalScale	the optimal scale identified by MGC

Author(s)

Eric Bridgeford

References

Youjin Lee, et al. "Network Dependence Testing via Diffusion Maps and Distance-Based Correlations." ArXiv (2019).

Examples

```
## Not run:
library(mgc)
library(MASS)

n = 100; d = 2
# simulate 100 samples, where first 50 have mean [0,0] and second 50 have mean [1,1]
Y <- c(replicate(n/2, 0), replicate(n/2, 1))
X <- do.call(rbind, lapply(Y, function(y) {
  return(rnorm(d) + y)
}))
# p value is small
mgc.ksample(X, Y, mgc.opts=list(nperm=100))$p.value

## End(Not run)
```

mgc.localcorr

MGC Local Correlations

Description

Compute all local correlation coefficients in $O(n^2 \log n)$

Usage

```
mgc.localcorr(
  X,
  Y,
  is.dist.X = FALSE,
  dist.xfm.X = mgc.distance,
  dist.params.X = list(method = "euclidean"),
  dist.return.X = NULL,
  is.dist.Y = FALSE,
  dist.xfm.Y = mgc.distance,
  dist.params.Y = list(method = "euclidean"),
  dist.return.Y = NULL,
  option = "mgc"
)
```

Arguments

X	<p>is interpreted as:</p> <p>a [n x d] data matrix X is a data matrix with n samples in d dimensions, if flag <code>is.dist.X=FALSE</code>.</p> <p>a [n x n] distance matrix X is a distance matrix. Use flag <code>is.dist.X=TRUE</code>.</p>
Y	<p>is interpreted as:</p> <p>a [n x d] data matrix Y is a data matrix with n samples in d dimensions, if flag <code>is.dist.Y=FALSE</code>.</p> <p>a [n x n] distance matrix Y is a distance matrix. Use flag <code>is.dist.Y=TRUE</code>.</p>
<code>is.dist.X</code>	a boolean indicating whether your X input is a distance matrix or not. Defaults to <code>FALSE</code> .
<code>dist.xfm.X</code>	if <code>is.dist == FALSE</code> , a distance function to transform X. If a distance function is passed, it should accept an [n x d] matrix of n samples in d dimensions and return a [n x n] distance matrix as the <code>\$D</code> return argument. See mgc.distance for details.
<code>dist.params.X</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm.X</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return.X</code>	<p>the return argument for the specified <code>dist.xfm.X</code> containing the distance matrix. Defaults to <code>FALSE</code>.</p> <p><code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm</code> as the distance matrix. Should be a [n x n] matrix.</p> <p><code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm.X[[dist.return]]</code> as the distance matrix. Should be a [n x n] matrix.</p>
<code>is.dist.Y</code>	a boolean indicating whether your Y input is a distance matrix or not. Defaults to <code>FALSE</code> .
<code>dist.xfm.Y</code>	if <code>is.dist == FALSE</code> , a distance function to transform Y. If a distance function is passed, it should accept an [n x d] matrix of n samples in d dimensions and return a [n x n] distance matrix as the <code>dist.return.Y</code> return argument. See mgc.distance for details.
<code>dist.params.Y</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm.Y</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return.Y</code>	<p>the return argument for the specified <code>dist.xfm.Y</code> containing the distance matrix. Defaults to <code>FALSE</code>.</p> <p><code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm.Y(Y)</code> as the distance matrix. Should be a [n x n] matrix.</p> <p><code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm.Y(Y)[[dist.return]]</code> as the distance matrix. Should be a [n x n] matrix.</p>
<code>option</code>	<p>is a string that specifies which global correlation to build up-on. Defaults to <code>'mgc'</code>.</p> <p>'mgc' use the MGC global correlation.</p> <p>'dcor' use the dcor global correlation.</p> <p>'mantel' use the mantel global correlation.</p> <p>'rank' use the rank global correlation.</p>

Value

A list contains the following:

corr	consists of all local correlations within [-1,1] by double matrix index
varX	contains all local variances for X.
varY	contains all local variances for X.

Author(s)

C. Shen

Examples

```
library(mgc)

n=200; d=2
data <- mgc.sims.linear(n, d)
lcor <- mgc.localcorr(data$X, data$Y)
```

mgc.localcorr.driver *Driver for MGC Local Correlations*

Description

Driver for MGC Local Correlations

Usage

```
mgc.localcorr.driver(DX, DY, option = "mgc")
```

Arguments

DX	the first distance matrix.
DY	the second distance matrix.
option	is a string that specifies which global correlation to build up-on. Defaults to 'mgc'. 'mgc' use the MGC global correlation. 'dcor' use the dcor global correlation. 'mantel' use the mantel global correlation. 'rank' use the rank global correlation.

Value

A list contains the following:

corr	consists of all local correlations within [-1,1] by double matrix index
varX	contains all local variances for X.
varY	contains all local variances for X.

Author(s)

C. Shen

mgc.sims.2ball	<i>Sample from Unit 2-Ball</i>
----------------	--------------------------------

Description

Sample from the 2-ball in d-dimensions.

Usage

```
mgc.sims.2ball(n, d, r = 1, cov.scale = 0)
```

Arguments

n	the number of samples.
d	the number of dimensions.
r	the radius of the 2-ball. Defaults to 1.
cov.scale	if desired, sample from 2-ball with error sigma. Defaults to NaN, which has no noise.

Value

the points sampled from the ball, as a [n, d] array.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
# sample 100 points from 3-d 2-ball with radius 2
X <- mgc.sims.2ball(100, 3, 2)
```

mgc.sims.2sphere *Sample from Unit 2-Sphere*

Description

Sample from the 2-sphere in d-dimensions.

Usage

```
mgc.sims.2sphere(n, d, r, cov.scale = 0)
```

Arguments

n	the number of samples.
d	the number of dimensions.
r	the radius of the 2-ball. Defaults to 1.
cov.scale	if desired, sample from 2-ball with error sigma. Defaults to 0, which has no noise.

Value

the points sampled from the sphere, as a [n, d] array.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
# sample 100 points from 3-d 2-sphere with radius 2
X <- mgc.sims.2sphere(100, 3, 2)
```

mgc.sims.cubic *Cubic Simulation*

Description

A function for Generating a cubic simulation.

Usage

```
mgc.sims.cubic(
  n,
  d,
  eps = 80,
  ind = FALSE,
  a = -1,
  b = 1,
  c.coef = c(-12, 48, 128),
  s = 1/3
)
```

Arguments

n the number of samples for the simulation.

d the number of dimensions for the simulation setting.

eps the noise level for the simulation. Defaults to 80.

ind whether to sample x and y independently. Defaults to FALSE.

a the lower limit for the range of the data matrix. Defaults to -1.

b the upper limit for the range of the data matrix. Defaults to 1.

c.coef the coefficients for the cubic function, where the first value is the first order coefficient, the second value the quadratic coefficient, and the third the cubic coefficient. Defaults to $c(-12, 48, 128)$.

s the scaling for the center of the cubic. Defaults to $1/3$.

Value

a list containing the following:

X $[n, d]$ the data matrix with n samples in d dimensions.

Y $[n]$ the response array.

Details

Given: $w_i = \frac{1}{i}$ is a weight-vector that scales with the dimensionality. Simulates n points from $Linear(X, Y) \in \mathbf{R}^d \times \mathbf{R}$, where:

$$X \sim U(a, b)^d$$

$$Y = c_3 (w^T X - s)^3 + c_2 (w^T X - s)^2 + c_1 (w^T X - s) + \kappa \epsilon$$

and $\kappa = 1$ if $d = 1$, and 0 otherwise controls the noise for higher dimensions.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.cubic(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.exp

*Exponential Simulation***Description**

A function for Generating an exponential simulation.

Usage

```
mgc.sims.exp(n, d, eps = 10, ind = FALSE, a = 0, b = 3)
```

Arguments

n the number of samples for the simulation.
d the number of dimensions for the simulation setting.
eps the noise level for the simulation. Defaults to 10.
ind whether to sample x and y independently. Defaults to FALSE.
a the lower limit for the range of the data matrix. Defaults to 0.
b the upper limit for the range of the data matrix. Defaults to 3.

Value

a list containing the following:

X [n, d] the data matrix with n samples in d dimensions.
Y [n] the response array.

Details

Given: $w_i = \frac{1}{i}$ is a weight-vector that scales with the dimensionality. Simulates n points from $Linear(X, Y) \in \mathbf{R}^d \times \mathbf{R}$, where:

$$X \sim U(a, b)^d$$

$$Y = e^{w^T X} + \kappa \epsilon$$

and $\kappa = 1$ if $d = 1$, and 0 otherwise controls the noise for higher dimensions.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.exp(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.joint *Joint Normal Simulation*

Description

A function for Generating a joint-normal simulation.

Usage

```
mgc.sims.joint(n, d, eps = 0.5)
```

Arguments

n the number of samples for the simulation.
d the number of dimensions for the simulation setting.
eps the noise level for the simulation. Defaults to 0.5.

Value

a list containing the following:

X [n, d] the data matrix with n samples in d dimensions.
Y [n] the response array.

Details

Given: $\rho = \frac{1}{2}d$, I_d is the identity matrix of size $d \times d$, J_d is the matrix of ones of size $d \times d$.
Simulates n points from *Joint - Normal* $(X, Y) \in \mathbf{R}^d \times \mathbf{R}^d$, where:

$$(X, Y) \sim N(0, \Sigma)$$

,

$$\Sigma = [I_d, \rho J_d; \rho J_d, (1 + \epsilon\kappa)I_d]$$

and $\kappa = 1$ if $d = 1$, and 0 otherwise controls the noise for higher dimensions.

For more details see the help vignette: `vignette("sims", package = "mgc")`

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.joint(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.linear *Linear Simulation*

Description

A function for Generating a linear simulation.

Usage

```
mgc.sims.linear(n, d, eps = 1, ind = FALSE, a = -1, b = 1)
```

Arguments

n	the number of samples for the simulation.
d	the number of dimensions for the simulation setting.
eps	the noise level for the simulation. Defaults to 1.
ind	whether to sample x and y independently. Defaults to FALSE.
a	the lower limit for the range of the data matrix. Defaults to -1.
b	the upper limit for the range of the data matrix. Defaults to 1.

Value

a list containing the following:

X	[n, d] the data matrix with n samples in d dimensions.
Y	[n] the response array.

Details

Given: $w_i = \frac{1}{i}$ is a weight-vector that scales with the dimensionality. Simulates n points from $Linear(X, Y) \in \mathbf{R}^d \times \mathbf{R}$, where:

$$X \sim U(a, b)^d$$

$$Y = w^T X + \kappa \epsilon$$

and $\kappa = 1$ if $d = 1$, and 0 otherwise controls the noise for higher dimensions.

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.linear(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.quad

*Quadratic Simulation***Description**

A function for Generating a quadratic simulation.

Usage

```
mgc.sims.quad(n, d, eps = 0.5, ind = FALSE, a = -1, b = 1)
```

Arguments

n the number of samples for the simulation.
d the number of dimensions for the simulation setting.
eps the noise level for the simulation. Defaults to 0.5.
ind whether to sample x and y independently. Defaults to FALSE.
a the lower limit for the data matrix. Defaults to -1.
b the upper limit for the data matrix. Defaults to 1.

Value

a list containing the following:

X [n, d] the data matrix with n samples in d dimensions.
Y [n] the response array.

Details

Given: $w_i = \frac{1}{i}$ is a weight-vector that scales with the dimensionality. Simulates n points from $Quadratic(X, Y) \in \mathbf{R}^d \times \mathbf{R}$ where:

$$X \sim U(a, b)^d$$

,

$$Y = (w^T X)^2 + \kappa \epsilon N(0, 1)$$

and $\kappa = 1$ if $d = 1$, and 0 otherwise controls the noise for higher dimensions.

For more details see the help vignette: `vignette("sims", package = "mgc")`

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.quad(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.spiral *Spiral Simulation*

Description

A function for Generating a spiral simulation.

Usage

```
mgc.sims.spiral(n, d, eps = 0.4, a = 0, b = 5)
```

Arguments

n the number of samples for the simulation.
d the number of dimensions for the simulation setting.
eps the noise level for the simulation. Defaults to 0.5.
a the lower limit for the data matrix. Defaults -1.
b the upper limit for the data matrix. Defaults to 1.

Value

a list containing the following:

X [n, d] the data matrix with n samples in d dimensions.
Y [n] the response array.

Details

Given: $U \sim U(a, b)$ a random variable. Simulates n points from $Spiral(X, Y) \in \mathbf{R}^d \times \mathbf{R}$ where:
 $X_i = U \cos(\pi U)^d$ if $i = d$, and $U \sin(\pi U) \cos^i(\pi U)$ otherwise

$$Y = U \sin(\pi U) + \epsilon p N(0, 1)$$

For more details see the help vignette: `vignette("sims", package = "mgc")`

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.spiral(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.step

*Step Function Simulation***Description**

A function for Generating a step function simulation.

Usage

```
mgc.sims.step(n, d, eps = 1, ind = FALSE, a = -1, b = 1)
```

Arguments

`n` the number of samples for the simulation.
`d` the number of dimensions for the simulation setting.
`eps` the noise level for the simulation. Defaults to 1.
`ind` whether to sample x and y independently. Defaults to FALSE.
`a` the lower limit for the data matrix. Defaults to -1.
`b` the upper limit for the data matrix. Defaults to 1.

Value

a list containing the following:

`X` [n, d] the data matrix with n samples in d dimensions.
`Y` [n] the response array.

Details

Given: $w_i = \frac{1}{i}$ is a weight-vector that scales with the dimensionality. Simulates n points from $Step(X, Y) \in \mathbf{R}^d \times \mathbf{R}$ where:

$$X \sim U(a, b)^d$$

,

$$Y = \mathbf{I}\{w^T X > 0\} + \kappa \epsilon N(0, 1)$$

and $\kappa = 1$ if $d = 1$, and 0 otherwise controls the noise for higher dimensions.

For more details see the help vignette: `vignette("sims", package = "mgc")`

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.step(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.ubern *Uncorrelated Bernoulli Simulation*

Description

A function for Generating an uncorrelated bernoulli simulation.

Usage

```
mgc.sims.ubern(n, d, eps = 0.5, p = 0.5)
```

Arguments

n the number of samples for the simulation.
d the number of dimensions for the simulation setting.
eps the noise level for the simulation. Defaults to 0.5.
p the bernoulli probability.

Value

a list containing the following:

X [n, d] the data matrix with n samples in d dimensions.
Y [n] the response array.

Details

Given: $w_i = \frac{1}{i}$ is a weight-vector that scales with the dimensionality. Simulates n points from $Wshape(X, Y) \in \mathbf{R}^d \times \mathbf{R}$ where:

$$U \sim \text{Bern}(p)$$

$$X \sim \text{Bern}(p)^d + \epsilon N(0, I_d)$$

$$Y = (2U - 1)w^T X + \epsilon N(0, 1)$$

For more details see the help vignette: `vignette("sims", package = "mgc")`

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.ubern(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.sims.wshape *W Shaped Simulation*

Description

A function for Generating a W-shaped simulation.

Usage

```
mgc.sims.wshape(n, d, eps = 0.5, ind = FALSE, a = -1, b = 1)
```

Arguments

`n` the number of samples for the simulation.
`d` the number of dimensions for the simulation setting.
`eps` the noise level for the simulation. Defaults to 0.5.
`ind` whether to sample `x` and `y` independently. Defaults to FALSE.
`a` the lower limit for the data matrix. Defaults -1.
`b` the upper limit for the data matrix. Defaults to 1.

Value

a list containing the following:

`X` [`n`, `d`] the data matrix with `n` samples in `d` dimensions.
`Y` [`n`] the response array.

Details

Given: $w_i = \frac{1}{i}$ is a weight-vector that scales with the dimensionality. Simulates n points from W -shape $(X, Y) \in \mathbf{R}^d \times \mathbf{R}$ where:

$$U \sim U(a, b)^d$$

,

$$X \sim U(a, b)^d$$

,

$$Y = \left[\left((w^T X)^2 - \frac{1}{2} \right)^2 + \frac{w^T U}{500} \right] + \kappa \epsilon N(0, 1)$$

and $\kappa = 1$ if $d = 1$, and 0 otherwise controls the noise for higher dimensions.

For more details see the help vignette: `vignette("sims", package = "mgc")`

Author(s)

Eric Bridgeford

Examples

```
library(mgc)
result <- mgc.sims.wshape(n=100, d=10) # simulate 100 samples in 10 dimensions
X <- result$X; Y <- result$Y
```

mgc.stat

MGC Test

Description

The main function that computes the MGC measure between two datasets: It first computes all local correlations, then use the maximal statistic among all local correlations based on thresholding.

Usage

```
mgc.stat(
  X,
  Y,
  is.dist.X = FALSE,
  dist.xfm.X = mgc.distance,
  dist.params.X = list(method = "euclidean"),
  dist.return.X = NULL,
  is.dist.Y = FALSE,
  dist.xfm.Y = mgc.distance,
  dist.params.Y = list(method = "euclidean"),
  dist.return.Y = NULL,
  option = "mgc"
)
```

Arguments

X	is interpreted as: a [n x d] data matrix X is a data matrix with n samples in d dimensions, if flag <code>is.dist.X=FALSE</code> . a [n x n] distance matrix X is a distance matrix. Use flag <code>is.dist.X=TRUE</code> .
Y	is interpreted as: a [n x d] data matrix Y is a data matrix with n samples in d dimensions, if flag <code>is.dist.Y=FALSE</code> . a [n x n] distance matrix Y is a distance matrix. Use flag <code>is.dist.Y=TRUE</code> .
<code>is.dist.X</code>	a boolean indicating whether your X input is a distance matrix or not. Defaults to <code>FALSE</code> .
<code>dist.xfm.X</code>	if <code>is.dist == FALSE</code> , a distance function to transform X. If a distance function is passed, it should accept an [n x d] matrix of n samples in d dimensions and return a [n x n] distance matrix as the \$D return argument. See mgc.distance for details.

<code>dist.params.X</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm.X</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return.X</code>	the return argument for the specified <code>dist.xfm.X</code> containing the distance matrix. Defaults to <code>FALSE</code> . <code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm</code> as the distance matrix. Should be a <code>[n x n]</code> matrix. <code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm.X[[dist.return]]</code> as the distance matrix. Should be a <code>[n x n]</code> matrix.
<code>is.dist.Y</code>	a boolean indicating whether your <code>Y</code> input is a distance matrix or not. Defaults to <code>FALSE</code> .
<code>dist.xfm.Y</code>	if <code>is.dist == FALSE</code> , a distance function to transform <code>Y</code> . If a distance function is passed, it should accept an <code>[n x d]</code> matrix of <code>n</code> samples in <code>d</code> dimensions and return a <code>[n x n]</code> distance matrix as the <code>dist.return.Y</code> return argument. See mgc.distance for details.
<code>dist.params.Y</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm.Y</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return.Y</code>	the return argument for the specified <code>dist.xfm.Y</code> containing the distance matrix. Defaults to <code>FALSE</code> . <code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm.Y(Y)</code> as the distance matrix. Should be a <code>[n x n]</code> matrix. <code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm.Y(Y)[[dist.return]]</code> as the distance matrix. Should be a <code>[n x n]</code> matrix.
<code>option</code>	is a string that specifies which global correlation to build up-on. Defaults to <code>'mgc'</code> . <code>'mgc'</code> use the MGC global correlation. <code>'dcor'</code> use the dcor global correlation. <code>'mantel'</code> use the mantel global correlation. <code>'rank'</code> use the rank global correlation.

Value

A list containing the following:

<code>stat</code>	is the sample MGC statistic within <code>[-1, 1]</code>
<code>localCorr</code>	the local correlations
<code>optimalScale</code>	the optimal scale identified by MGC
<code>option</code>	specifies which global correlation was used

Author(s)

C. Shen and Eric Bridgeford

References

Joshua T. Vogelstein, et al. "Discovering and deciphering relationships across disparate data modalities." *eLife* (2019).

Examples

```
library(mgc)

n=200; d=2
data <- mgc.sims.linear(n, d)
mgc.stat.res <- mgc.stat(data$X, data$Y)
```

mgc.test

MGC Permutation Test

Description

Test of Dependence using MGC Approach.

Usage

```
mgc.test(
  X,
  Y,
  is.dist.X = FALSE,
  dist.xfm.X = mgc.distance,
  dist.params.X = list(method = "euclidean"),
  dist.return.X = NULL,
  is.dist.Y = FALSE,
  dist.xfm.Y = mgc.distance,
  dist.params.Y = list(method = "euclidean"),
  dist.return.Y = NULL,
  nperm = 1000,
  option = "mgc",
  no_cores = 1
)
```

Arguments

X	is interpreted as: a [n x d] data matrix X is a data matrix with n samples in d dimensions, if flag <code>is.dist.X=FALSE</code> . a [n x n] distance matrix X is a distance matrix. Use flag <code>is.dist.X=TRUE</code> .
Y	is interpreted as: a [n x d] data matrix Y is a data matrix with n samples in d dimensions, if flag <code>is.dist.Y=FALSE</code> . a [n x n] distance matrix Y is a distance matrix. Use flag <code>is.dist.Y=TRUE</code> .
<code>is.dist.X</code>	a boolean indicating whether your X input is a distance matrix or not. Defaults to FALSE.

<code>dist.xfm.X</code>	if <code>is.dist == FALSE</code> , a distance function to transform X. If a distance function is passed, it should accept an $[n \times d]$ matrix of n samples in d dimensions and return a $[n \times n]$ distance matrix as the <code>\$D</code> return argument. See mgc.distance for details.
<code>dist.params.X</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm.X</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return.X</code>	the return argument for the specified <code>dist.xfm.X</code> containing the distance matrix. Defaults to <code>FALSE</code> . <code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm</code> as the distance matrix. Should be a $[n \times n]$ matrix. <code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm.X[[dist.return]]</code> as the distance matrix. Should be a $[n \times n]$ matrix.
<code>is.dist.Y</code>	a boolean indicating whether your Y input is a distance matrix or not. Defaults to <code>FALSE</code> .
<code>dist.xfm.Y</code>	if <code>is.dist == FALSE</code> , a distance function to transform Y. If a distance function is passed, it should accept an $[n \times d]$ matrix of n samples in d dimensions and return a $[n \times n]$ distance matrix as the <code>dist.return.Y</code> return argument. See mgc.distance for details.
<code>dist.params.Y</code>	a list of trailing arguments to pass to the distance function specified in <code>dist.xfm.Y</code> . Defaults to <code>list(method='euclidean')</code> .
<code>dist.return.Y</code>	the return argument for the specified <code>dist.xfm.Y</code> containing the distance matrix. Defaults to <code>FALSE</code> . <code>is.null(dist.return)</code> use the return argument directly from <code>dist.xfm.Y(Y)</code> as the distance matrix. Should be a $[n \times n]$ matrix. <code>is.character(dist.return) is.integer(dist.return)</code> use <code>dist.xfm.Y(Y)[[dist.return]]</code> as the distance matrix. Should be a $[n \times n]$ matrix.
<code>nperm</code>	specifies the number of replicates to use for the permutation test. Defaults to 1000.
<code>option</code>	is a string that specifies which global correlation to build up-on. Defaults to <code>'mgc'</code> . <code>'mgc'</code> use the MGC global correlation. <code>'dcor'</code> use the dcor global correlation. <code>'mantel'</code> use the mantel global correlation. <code>'rank'</code> use the rank global correlation.
<code>no_cores</code>	the number of cores to use for the permutations. Defaults to 1.

Value

A list containing the following:

<code>p.value</code>	P-value of MGC
<code>stat</code>	is the sample MGC statistic within $[-1, 1]$
<code>p.localCorr</code>	P-value of the local correlations by double matrix index.
<code>localCorr</code>	the local correlations
<code>optimalScale</code>	the optimal scale identified by MGC
<code>option</code>	specifies which global correlation was used

Details

A test of independence using the MGC approach, described in Vogelstein et al. (2019). For $X \sim F_X, Y \sim F_Y$:

$$H_0 : F_X \neq F_Y$$

and:

$$H_A : F_X = F_Y$$

Note that one should avoid report positive discovery via minimizing individual p-values of local correlations, unless corrected for multiple hypotheses.

For details on usage see the help vignette: `vignette("mgc", package = "mgc")`

Author(s)

Eric Bridgeford and C. Shen

References

Joshua T. Vogelstein, et al. "Discovering and deciphering relationships across disparate data modalities." eLife (2019).

Examples

```
## Not run:
library(mgc)

n = 100; d = 2
data <- mgc.sims.linear(n, d)
# note: on real data, one would put nperm much higher (at least 100)
# nperm is set to 10 merely for demonstration purposes
result <- mgc.test(data$X, data$Y, nperm=10)

## End(Not run)
```

Index

ConnCompLabel, [2](#)

discr.sims.cross, [3](#)

discr.sims.exp, [4](#)

discr.sims.fat_tails, [5](#)

discr.sims.linear, [6](#)

discr.sims.radial, [7](#)

discr.stat, [8](#)

discr.test.one_sample, [10](#)

discr.test.two_sample, [12](#)

dist, [15](#)

mgc.dist.xfm, [14](#)

mgc.distance, [9](#), [11](#), [13](#), [15](#), [18](#), [31](#), [32](#), [34](#)

mgc.ksample, [16](#)

mgc.localcorr, [17](#)

mgc.localcorr.driver, [19](#)

mgc.sims.2ball, [20](#)

mgc.sims.2sphere, [21](#)

mgc.sims.cubic, [21](#)

mgc.sims.exp, [23](#)

mgc.sims.joint, [24](#)

mgc.sims.linear, [25](#)

mgc.sims.quad, [26](#)

mgc.sims.spiral, [27](#)

mgc.sims.step, [28](#)

mgc.sims.ubern, [29](#)

mgc.sims.wshape, [30](#)

mgc.stat, [31](#)

mgc.test, [16](#), [33](#)